Individualism and Collectivism: Examining Student Mathematical Identity in Hierarchal Grouping Arrangements

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This paper is an investigation into the impact of a hierarchal and exclusive ability grouping framework on the self-perception and mathematical identity of diverse students in Aotearoa, New Zealand. Research interest in the low achievement of certain groups of students is growing as a result of increased understanding of the impact of cultural and societal norms. However, there has been less research into factors that influence how to support student perception to promote effective learning. This case study examines how perceptions of ability can be viewed as either exclusive or inclusive, and how this can be used to capture and cater to the complexity of a student's identity as a mathematician.

Diversity is defined within this paper as all students who are marginalised due to ethnicity, class, sex, disability, status, cultural, or social identity. One of the challenges in implementing equitable grouping arrangements is changing student perception of what maths looks like and what it means to succeed as a mathematician. There is a distinctive and fixed culture in Aotearoa mathematics classrooms which promotes a linear hierarchy, both in terms of achievement and status (Averill, 2018; Hunter & Hunter, 2018; Milne 2017). Equity-promoting professional learning development (PLD) projects, such as Developing Mathematical Inquiry Communities (DMIC), seek to address and challenge this hierarchy. As part of the baseline data collection at the beginning of the PLD, students' self-perception of mathematics teaching and learning was investigated via a survey and an interview.

The hierarchy of mathematical ability is a microcosm of the greater social hierarchies that exist in our society. Race, sex, class, disability and other identities are marginalised and adversely placed in the social and academic hierarchies of the mathematics classroom in mirror image as to how they exist in the macrocosm societal hierarchies (Louie, 2017). As part of addressing these inequities in Aotearoa, New Zealand, educators, researchers, and policy makers have focused on culturally responsive teaching initiatives. Unfortunately, some of these initiatives have been unsuccessful due to the dominant framework which is so deeply embedded in policy, history, and societal norms (Averill, 2018; Milne 2017). Teachers are embedded in the exclusive framework, the same framework they learnt in as students themselves. Often, they are also members of the hegemonic ethnicity, class, and culture. Hunter and Hunter (2018) refer to this as a "pedagogical challenge" for teachers who are attempting to address inequity.

Literature Review

A culture of individualism has prevailed in Aotearoa, New Zealand classrooms since formal schooling began here, based on a Euro-centric approach. Milne's (2017) research reports bleak findings around the hegemonic Pakeha culture in classrooms, where other cultures are marginalised almost to the point of cultural extinction. Milne refers to a "white background," which is considered as "nothing," or "blank." In reality, rather than being "nothing," it is everything. It is the reinforcement of "pre-determined boundaries and expectations," designed to "white-stream" students into a uniform western cultural ideology, which by its very nature, excludes all who do not align with it. In more recent times, there has been a push, both nationally and internationally, for a more collective approach to

2019. In G. Hine, S. Blackley, & A. Cooke (Eds.). Mathematics Education Research: Impacting Practice *(Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia)* pp. 372-379. Perth: MERGA.

teaching and learning across all curriculum areas. These reforming transitions have been instigated from the governmental level via the New Zealand Ministry of Education. However, this culture of exclusion which exists in the DNA of traditional classrooms must be examined as the root of our pedagogical beliefs and approaches in order to eradicate the cause of exclusion and create the conditions for inclusion to flourish (Barkatsas, 2012; Cohen & Lotan, 1997; Louie, 2017).

The linear hierarchal structure around both ability and tasks in the mathematics classroom provides a fixed and narrow trajectory along which all students must travel (Louie, 2017). This trajectory creates an inherent ideology of consistently inequitable achievement which appears "natural" and "normal," with some students occupying the bottom strata, while others necessarily exist at the top. Durkheim (1895) argues that exclusion is so deeply entrenched in western culture that both our behaviours and beliefs are interpreted as natural states of being as opposed to a social construct. This inequity can be seen in students who are always in the "low" group for maths and are repeatedly given "easy and boring" tasks. Stein and Henningsen (1997) claim that students come to view mathematics as a set of procedures and answers given out by an authority (the teacher or textbook), and view themselves in the role of memoriser, rather than a "doer" of mathematics. They perceive their inadequacies as fixed, normal, and natural. This has adverse consequences in terms of their opportunities for achievement, which are already disproportionally stacked against them (Cohen & Lotan, 1997; Ensign, 2003; Louie, 2017).

Louie's (2017) research supports this deliberate and methodical approach through her analysis of exclusion in a large urban high school Mathematics Department in Western United States. Louie developed a table of four frames (Exclusionary/ Inclusive Practice and Hierarchal/Multi-dimensional ability) through which to analyse the teaching and learning across four mathematics classrooms during an equity reforming PLD. Louie found that, although the teachers in her study were dedicated to the improvement of equity in their classrooms, their practice remained rooted in exclusionary frames (Louie, 2017). Her findings are consistent with Lawler's (2018) study which reported teachers to still be viewed as the fount, the "downloader," and the validator of knowledge, despite many modern reforms to education.

Part of the exclusionary frame for student self-perception is a belief that their culture exists outside the classroom, particularly the mathematics classroom. Often there is a mismatch between the culture of home and the classroom, especially for Pasifika students, who value "reciprocity, family, relationships, spirituality, leadership, collectivism, love, and belonging," (Hunter & Hunter, 2018). Asai and Lucca (1998) claim that the prevailing western classroom culture, in antithesis, values competition, formal relationships, secularity, individualism, and a uniform adherence to the status quo. Hunter and Hunter (2018) discuss the importance of bringing the values of collectivism into the mathematics classroom. They use the analogies of a 'waka,' (canoe) where all students row together, and making a 'siapo,' (a woven mat), which is made collectively but with individual skills. The definition of culture, however, is not restricted to ethnicity but expands to encompass all aspects of culture: religion, class, beliefs, values and lifestyle choices. All families have their own unique culture and the value of this has largely been overlooked in educational settings. The research of Ishimaru and colleagues (2015) supports these findings by suggesting that mathematics in routine parts of family culture are currently invisible in classrooms and, in order to uphold the value of "epistemic authority" and build on student mathematical identity, we must find a way of making them "visible." Jurdak (2009) claims that visibility of diverse cultures is imperative because diverse family cultures differ from the *status quo* in a way which serves to exclude them from the opportunities they need to achieve.

Research Methods

Background

This study took place in a public, single-sex high school in a small city in Aotearoa, New Zealand. The school houses both day students and boarding house residents, with an overall population of 1198 students. The ethnicity demographic is 783 NZ European/ 'Pakeha,' 320 Maori, 40 Pasifika, 20 Asian and 5 other. The mathematics classes are ability-streamed. Participation in the DMIC project was instigated by one teacher in the mathematics department who initiated the PLD. There was a strong commitment amongst some of the department to improve teaching for equity and lift student achievement. All Year 9 and 10 (ages 13 - 14) mathematics students participated in the project, with a sample size of 51 students participating in the survey and interview process. A cross-section of year levels, ability streams, and ethnicities were represented. The survey and interview process took place over the course of several weeks early in the academic year.

Data Collection and Analysis

This small case study emerged from a larger scale project (the DMIC project). It draws on baseline data collected from initial interviews and surveys. The interviews were audio-recorded and transcribed at a later date. Students completed a twelve-statement values survey before the interview was conducted. Students were asked to discuss their responses to the survey questionnaire before a set of further questions were asked. These questions were designed to gauge their perception of mathematics teaching and learning, as well as beliefs around grouping arrangements, task design, content, and the purpose of learning.

The data were analysed via a tally of score totals before exploration commenced into emerging themes which surfaced through NVivo Projects. The fifty-one interviews were divided into three separate NVivo projects and coded according to pre-established criteria by three researchers. The researchers met via Zoom online and co-coded selected interviews, taking note of any discrepancies in criteria for discussion between the whole research team. Once completed, the research team met again to discuss the findings. Initial findings from data and discussions generated an investigation by one researcher into one specific theme which emerged, and this formed the basis for the case study.

Analysis was undertaken through a focused investigation. Comparisons to the survey data confirmed a similar analysis to the interview data. The researcher generated a baseline data summary report. Upon fuller immersion into the data, the investigation became grounded in Nicole Louie's theory of exclusionary framing. Louie's framework provided a clear and concise method of organising the findings as shown in Table 1.

Table 1
Ways of Framing Mathematics and Mathematical Ability

Exclusionary	Inclusive
The rote practice frame *Mathematics is a fixed body of knowledge to be absorbed and practiced. Correctness is paramount.* • Presenting standard formulas, algorithms, and so forth • Assigning routine tasks requiring only the application of previously demonstrated algorithms • Asking closed questions in conversation with students • Explicitly stating the importance of repetitive practice • Focusing discussion exclusively on answers	***The sense making frame*** *Mathematics is about making sense of ideas and understanding connections:* • Assigning open-ended, non-routine tasks • Asking open-ended questions and pressing for meaning in conversation with students • Explicitly stating the importance of sense making ***The multidimensional maths frame*** *Mathematics includes activities such as collaboration, experimentation, and argumentation, not just rote practice.* • Assigning open-ended, non-routine tasks • Explicitly naming skills that have not traditionally been seen as mathematical as mathematically important
The hierarchical ability frame *Mathematical ability is distributed along a linear continuum. Some people have a lot; others have very little.* • Explicitly valorising speed and correctness • Positioning some students as helpers and others as in need of help	***The multidimensional ability frame*** *Everyone has both intellectual strengths and* *Areas for growth that are relevant to mathematics learning.* • Valorising skills that have not traditionally been seen as mathematical • Naming a variety of students as resources for their peers" learning • Making statements about mutual dependence (everyone contributes, everyone learns together).

Findings

Exclusionary Framing of Ethnicity

Twenty-four out of fifty-one students described their ethnicity as "normal." It was normalised in the classroom to the extent of neutrality. Eight students specifically stated that their ethnicity was unrelated to mathematics and it had no relevance to their learning. Three of the fifty-one students made specific comments about valuing their own ethnicity in the classroom. Four of the fifty-one students specifically mentioned valuing other's culture; all comments were generalised around classroom climate, rather than learning. Only one student spoke negatively about ethnicity but, apart from the seven students who valued culture, all students defined it in terms of what it *wasn't* as opposed to what it *was* ("we don't have any discrimination or anything like that"). This finding is consistent with Milne's (2017) concept of "white spaces" in which the prevailing classroom culture, environment, and content is so pervasively Pakeha students view it as a 'normalised nothing,' as opposed to the cultural benchmark against which they compare all other cultures. Students referred to racism only in terms of what it was not (not discriminating against someone based on their race), but never in terms of what it *was*: privileging Pakeha beliefs, values, language, and culture over other ethnicities' beliefs, values, language, and culture.

Exclusionary Framing of Culture

One of the highest scoring personal values in the survey was "family" but it rated as one of the least important mathematics values. Forty-five of the fifty-one students reported feeling close to their families and rated this above friendship, hobbies, sport and extra-curricular activities. However, when it came specifically to their mathematics education, only eleven out of fifty-one students felt that their families were important to their success or identity as a mathematician. Students of all ethnicities talked specifically about concepts of inclusion ("we like to do stuff together") and collective success ("we all help at docking time, Mum stops her work to help Dad and we all go out on the farm to get the job done"), but these family values were viewed as a separate entity from school life. This anomaly supports the research of Ishimaru and colleagues (2015), whose research showed that family values, experiences and knowledge become invisible in the classroom. Two students talked about valuing family communalism but felt that there was a separation preventing this; they expressed a desire to spend more time with their family.

Exclusionary Framing of Ability

Forty-nine of the fifty-one students reinforced the hierarchal framework for ability. Mathematical proficiency was viewed as fixed, regardless of whether the student perceived themselves as capable or incapable. Thirty of the fifty-one students commented specifically on their belief in a fixed hierarchy of ability, while two students believed it was flexible and multi-dimensional. This finding supports Durkheim's original (1895), Louie's (2017), and Stein and Henningsen's (1997) research that focuses on the effects of self-perception under a narrow and linear criterion for success. Drukheim's study into a hierarchal society, although very old now, still holds the same relevance for inequity within the classroom and within the structure of society at large today. In this framework other students were exclusively viewed in relation to their own positioning on the linear hierarchy, as either "dumber," or "smarter." The school ethos of a fixed, linear hierarchy of ability was reiterated throughout the interviews. Achievement was viewed by forty-nine out of the fifty-one

students through a fatalistic lens ("I just don't find maths hard. I'm just good at it, naturally"). The diverse students amongst the above statistic viewed success through a deficit fatalism ("I am where I belong…in the low stream").

Exclusionary Framing of Mathematical Activity

While family and peers were viewed as largely irrelevant, the teacher and the textbook were viewed as highly influential. Eighteen of the fifty-one students explicitly positioned their teacher as the ultimate authority, whereas two students mentioned the value of student autonomy. These findings are consistent with Lawler's (2018) findings, which focuses on the prevalence of the teacher locus of control and the prohibitive effect this has on student autonomy and agency. The discipline of mathematics, like ability, was viewed as memorisation of a fixed body of knowledge. Twenty-three students referred to this multiple times during their interview, while four students mentioned the importance of understanding the mathematical principles underlying the task they were undertaking. The most common response to the question "what would you wish for if you could have one wish for your mathematics education?" was for memorisation ("I want a calculator head!"). Forty-seven students talked about the value of utility in mathematics. They valued it in terms of what it meant in their current lives, their future professions and everyday financial literacy as an adult. They did not want to "waste time" learning complicated algorithms that would not serve them in some practical, meaningful way.

Discussion

The findings in this case study support other, similar studies, around diversity in Aotearoa, New Zealand classrooms (Averill, 2018; Hunter & Hunter, 2018; Milne, 2017) as well as classrooms in the wider, western society (Lawler, 2018; Louie, 2017; Stein & Henningsen, 1997). These findings also support wider sociological studies, such as Durkheim's (1895), which reflect societal hierarchies that enable the exclusion of diverse populations of people. The findings from this case study highlight a mathematics classroom culture, which is largely non-representative of family culture, ethnicity diversity, multi-dimensional capabilities, and skills. These findings reiterate the normalisation of exclusion to the extent that status and ability hierarchies exist inside student self-perception as an innate state of being, an irrefutable fact, rather than a socially constructed (western) ideology.

The juxtaposition between ingrained "school values" and personal values created anomalies within the findings. While students perceived success in school mathematics as being able to memorise formulas, achieve the correct answer quickly and demonstrate a full body of knowledge; they preferred it to be focused on relevant understandings needed in "real" life. Students accepted this misalignment of values as "normal" and adapted their self-perception into compartments of school/real life. The anomaly between what students perceive as success ("school maths," which is unrelated to culture, home-life or real life) and what they say is personally important ("real maths," which is related to their own, very unique, real life) is consistent with Hunter and Hunter's (2017) findings of the importance of culturally relevant tasks and problems as key to the positive and holistic identity of mathematics students in and out of the classroom. Hunter and Hunter's Ministry funded PLD project DMIC provides a longitudinal study into a wide cross section of Aotearoa, New Zealand mathematics classrooms. Some of the recurring findings, thus far, focus on the importance of relevant contexts for problems, culturally responsive pedagogy, and multi-dimensional tasks. All four frames of exclusion analysed in this case-study support these findings.

Implications

Exclusion of diverse students has been investigated in this case study through the perspective of individualism as an historical, social and western construct. Individualism is inherently exclusive and reproduces competition, linear and hierarchal definitions of ability, uniformity of ideas and a narrow criterion for success. The mathematics classroom as a microcosm, representative of the larger macro-culture of an individualistic, capitalist western society; is located within an analysis of a much wider cultural context (Cohen & Lotan, 1997; Louie, 2017). Student self-perception, which reinforces a culture of exclusion, does not exist in and of itself; it is informed by teachers, family, the wider community and society at large. This trickles down to student perception where students with low self-efficacy talk about feeling "dumb" and call other students "smarter." The culture of exclusion within this school has not emerged from within a local construction of individual teacher and student beliefs, behaviours, or knowledge but from a macro-culture system that exists in the societal norms of our hegemonic culture.

More research is needed to better understand the shifts in self-perception of diverse students who experience a change in classroom climate from an individualistic cultural norm to a more collective one. The anomaly of some of the findings around family values suggests that there is a discrepancy between home cultural values and the larger community values, even within the same ethnic groups. An investigation into the sub-cultures within the wider community would serve to further challenge the dominant cultural norm and support a transformation towards a more collective norm. Jurdak (2009) extolled the need to make these sub-cultures more "visible" to improve the opportunities for diverse students to succeed. The findings in this case-study support this claim and suggest, furthermore, that the self-perception of non-diverse students would, also, be improved by this visibility. The true re-culturing of mathematics education requires a holistic overhaul of the system which stifles every student's perception of status and ability.

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